Are the number words learnable?

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Abstract

It is widely held that children learn the concept of natural number by inductive inference from their knowledge of the first few numbers, helped by their ability to count. Rips et al. (2006) argue that this view is wrong, and I argue that their argument is wrong.

What do number words mean? If a panel of linguists, psychologists, and philosophers was convened, it is more than likely that they would soon converge on an answer along the following lines:

(1) The n-th number word denotes that property which a collection x has iff card(x) = n.

(Here "iff" is short for "if and only if", and "card(x)" is the number of individuals in x.) This answer is not unproblematic in every respect: in particular, the notion of "collection" raises some rather deep issues. But apart from that, (1) surely represents the majority view across disciplines.

Natural though it may seem, this consensus shouldn't be taken for granted, for there are perfectly reasonable alternatives. For example, one might view number words as being on a par with quantifiers like *all*, and for some time it was fashionable, in some circles at least, to construe *two* as "two or more". But such dissident views are becoming increasingly marginal (Geurts 2006).

Next question: How do children learn number words and the associated concepts? This question, too, has a standard answer, according to which the acquisition process involves three parts. First, children learn to recite the count words in sequence. Secondly, learning the meanings of the first few number words involves a special process, like "subitising", for example (Kaufman et al. 1949); this process enables the child to master (1) for the first three number words. But, between them, being able to reproduce the number words in canonical order and knowing what *one*, *two*, and *three* mean do not entail a grasp of the concept of natural number, so a third step is necessary: the child has to observe that the first three number concepts are the start of a progression which goes on forever; and here they are helped by the fact that they know how to count.

This sounds like a plausible story, but Rips et al. (2006) argue that it is flawed. More accurately, their claim is that the generalisation children are said to acquire in the third step *underdetermines* the concept of natural number. According to Rips et al., the generalisation goes as follows:

(2) If any given number word denotes a property which x has if card(x) = n, then the next number word denotes a property which x has if card(x) = n+1.

Rips et al.'s main point is that this is too weak. To show this, they conjure up the case of poor Jan, a three-year-old girl who is taught the following deviant interpretation of the number words by a "diabolical parent":

(3)
- The number words *zero*, *ten*, *twenty*, ... all denote that property which x has iff card(x) = 0 or 10 or 20 or ...
- The number words *one*, *eleven*, *twenty one*, ... all denote that property which x has iff card(x) = 1 or 11 or 21 or ...
- ...

The problem is that Jan's interpretation is consistent with the generalisation in (2): it preserves the linear structure mandated by (2), even though Jan's line of number words closes in on itself after *ten*, which is clearly incorrect. Hence, Rips et al. conclude, "the insight ... achieved in inferring (2) ... does not by itself yield an understanding of the natural numbers." (Rips et al. 2006: B54)

It is important to note that this argument is a *logical* one. It is not about the contingencies of lexical and conceptual learning, but rather about the information children acquire when they learn (2). (Thus the fact that Jan is forced into this perverse construal of the number words is beside the point.) The key ingredients of the argument are as follows:

(i) To begin with, Jan's understanding of the first three number words is in line with (1). (Call this Jan's "inductive base".)
(ii) However, she gives up this interpretation in favour of the one in (3).

(iii) It is reasonable to assume that (3) is the generalisation about number words that children, including Jan, acquire.

In the following I will argue that the premisses (ii) and (iii) are not justified: there is no reason why Jan should leave her inductive base and the inductive generalisation that Rips et al. are willing to grant is unreasonably weak, i.e. it is much weaker than what a child might plausibly hypothesise on the basis of his understanding of the first three number words.

Jan leaves her inductive base for no apparent reason

Rips et al.'s scenario is unrealistic in that it assumes that Jan's construal of the number words changes as she learns the generalisation in (2). Jan is supposed to find out the principle connecting the first three number words as interpreted in (1), and duly arrives at (2), but at the same time reinterprets the number words so as to bring them in line with (3). This is very strange. To explain why, consider the following simile, whose protagonist is three-year-old Fran (who in Rips et al.'s story got at least the number words right). She knows the meanings of the words *father* and *mother*, and is aware of the meaning of *parent*, as well:

(4) The word *parent* denotes a relation which holds between two individuals x and y iff x is the father or mother of y.

Armed with this knowledge, Fran sets about acquiring the meaning of *ancestor*, which she interprets, sensibly enough, as follows:

(5) The word *ancestor* denotes a relation which holds between two individuals x and y iff (i) x is a parent of y or (ii) x is an ancestor of z and z is a parent of y.

Testing Fran's knowledge, we find her saying that her parents are among her ancestors, as are her grandparents, her great-grandparents, etc. So far so good. However, Fran also insists that her teacher is one of her ancestors, although she admits that he is not even family. How come? Well, it turns out that in the process of learning (5), Fran changed her concept of parent:

(6) The word *parent* denotes a relation which holds between two individuals x and y iff (i) x is the father or mother of y or (ii) x is German.

Fran's teacher is German—that's why.

I trust that nobody would be tempted to argue that the case of Fran demonstrates that (5) underdetermines the meaning of *ancestor*, except in the trivial sense that an understanding of (5) presupposes that the meaning of *parent* is understood correctly. But then the same should hold for Jan, mutatis mutandis.

Jan's generalisation is too weak

Principle (2) is too weak. That is what Rips et al. say, too, but my claim is that it is unreasonably weak—"unreasonable" in the sense it dramatically falls short of what a child like Jan can plausibly infer from her inductive base. To see how weak it really is, consider the following interpretation scheme:

(7) All number words denote the property of being a collection.

This construal perverts the semantics of number words by stipulating that all number words have the same meaning, and it, too, is consistent with (2) and the other premisses Rips et al. are willing to grant. Hence, the inductive rule Rips et al. give Jan to work with is basically *trivial*.

Apparently, something went seriously wrong, and in my opinion it's this: Rips et al.'s rule (2) is much weaker than what a child might plausibly infer under the given circumstances. Suppose a child can recite the canonical sequence of number words and is able to correctly assign number words to collections of cardinality 1, 2, and 3. Such a child may plausibly be expected to arrive at the following rule:

(8) If any given number word denotes the property which x has iff card(x) = n, then the next number word denotes the property which x has iff card(x) = n+1.

To explain the difference between (2) and (8), suppose that the number word *three* denotes the property "card(x) = 3". If this is the case, both rules apply. Now suppose that *three* denotes the property "card(x) = 3 or 13 or 23 or ...": in this case (2) still applies, but (8) does not. In other words, (8) is restricted in its application to number words that denote properties of the form "card(x) = n", and thus excludes non-canonical interpretations like Jan's.

Young Marvin, yet another hypothetical three-year-old, has seen 12 fire engines, all of them red, before he concludes that fire engines are red *or* blue. This is an unlikely scenario, of course, and for an obvious reason: since all the fire engines he encountered were red, it would be irrational for Marvin to conjecture that fire engines are red or blue. The case of Rips et al.'s Jan is essentially the same. By hypothesis, Jan knows the meanings of the first three number words, so we may assume that she has come across a fair number of positive instances of *one*, *two*, and *three* in the form of singletons, duos, and trios of billiard balls, bicycles, bananas, or what have you (and perhaps she has been exposed to negative instances, as well). This being the case, it makes no sense for her to adopt Rips et al.'s rule instead of the one in (8), simply because it would be gratuitously weak.

Conclusion

It is true that Jan's understanding of the number words is seriously flawed, but this doesn't prove what Rips et al. want it to prove. What it shows, rather, is that Jan is pathetically bad at inductive reasoning in two ways: she weakens her inductive base—her knowledge of the first three number words—for no apparent reason at all, and she comes up with a generalisation that is practically empty.

References

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